ASMR CH

AWFUL SWISS MATHEMATICS RELOADED 2025

Time: MMMMMMMMMMMMMMDCCCLIX seconds	Not here
Difficulty: you can do it, trust	Not now
Points: every complete solution is worth 10.5 points	Not this

1. Let n be a positive integer. We have an $n \times n$ square, inside of which there are ~~(n~~ $(n+1)^2$ points. Prove that three of these points are the vertecies of a (possibly degenerate) triangle whose area is at most $\frac{1}{2}$.

2. Let ABC be a non-equilateral triangle with integer sidelengths. Let D be the midpoint of BC, E the midpoint of CA and G the centroid of ABC. Find the minimal possible perimeter of ABC such that DCEG is cyclic.

3. Find all functions f from the integers to the integers such that for any two integers a and b, the difference between b and the function value of a divides the difference between the square of the function value of a and the function value of the square of b.

4. Find all functions $f: C \to C$ such that, for all x,y in C, the following holds:

$$f(xf(y)) + f(x^2 + y) = f(x + y)x + f(f(y))$$

h a v e f u n